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# Investigating Density Scale Height in the Solar Corona

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ABSTRACT. In this paper, electron density distribution with radial distance in the solar corona have been investigated using an exponential representation. Selection and solution criteria used to determine very accurately the density scale height in the solar corona. From the slope we computed the density scale height ($H_o$) with the value of $1.46 \times 10^7$ meters, which is in a good agreement with that mentioned in the literature. From this slope we found also the value of the arbitrary reference height, which is 0.5.

## 1. Introduction

The determination of solar atmosphere parameters from spectral lines have been of considerable importance and interest for many years. Numerous amount of data have been utilized for this purpose using considerable amount of observations from different ground-based and space solar observatories. Some of which are OSO-7, Skylab, SMM, Ulysses, Yohkoh, SOHO and Trace and many others will be on the run in a few years time, e.g. Solar-B[1-6].

In this paper, we will use data obtained by a spectrometer known as Coronal Density Spectrometer (CDS) on The Solar and Heliospheric Observatory (SOHO). This satellite has a unique operating mode that could be considered as almost as an interactive solar observatory, where it provides live data display at stations like Goddard Space Flight Center. Soho has certainly provided the solar community with a tremendous amount of knowledge and information reaching about 2000 articles in journals, conferences and workshops[5,7-8]. This very useful knowledge and detailed information about our star spans a wide range of spectrum ranging from the interior, through the hot and dynamic atmosphere reaching the solar wind with its twelve different sophisticated instruments aimed for investigating the three previous goals. Five of them have been designated mainly for studying the solar atmosphere its structure and dynamic, e.g. SUMER, CDS, EIT, UVCS and LASCO.

Recent data taken by the CDS regarding electron density and temperature have provided new views about the coronal plasma. For example, at the transition region[9] it has been shown that, by using ratios of line intensities of ions such as O III, O IV and O V, a significant change in density inhomogeniety or the filling factor produces an intensity increase in quiet sun network by the order of 10-40% (or what is known as the EUV flashes). In the corona, on the other hand, CDS and SUMER have measured the electron temperature as a function of height above the limb in a polar coronal hole. The temperature has been found to increase from $0.8 \times 10^6$ K close to the limb, rising to a maximum of less than $1.0 \times 10^6$ K at 1.15 *Ro* (where *Ro* is the solar radius), then falling to around $0.4 \times 10^6$ K at 1.3 *Ro*. On the other hand, on the equator the temperature was found to increase constantly as a function of height, from about $1 \times 10^6$ K close to the limb to about $3 \times 10^6$ K at 1.3 *Ro*.

However, electron densities using the line intensity ratios have been studied extensively[6,9-12]. The CDS has significantly improved many of the past results in terms of spatial resolution, and diagnostics capabilities. In particular, it can provide measurements very close to the limb, at distances unavailable from the white light observations. It has been observed that there is an exponential decay of line intensities in the solar atmosphere as a function of radial distance. This information has been used to calculate density scale heights from lines formed at coronal temperature ranging from $9 \times 10^5$ to $2 \times 10^6$ K. Spectral line ratios of density sensitivity, e.g. Si X 356/347, have been used to derive an average density which is found to decrease from $7 \times 10^8$ cm$^{-3}$ near the limb to $1.5 \times 10^8$ cm$^{-3}$ at 1.2 *Ro*.

In the present paper, our investigation will concentrate on above the limb observations of the quiet sun in the coronal holes. This particularly involves measurements of line intensities as a function of the radial distance, where we will investigate the electron density scale height. In Section 2, we discussed the model for the electron density, where we have chosen a simple one for this purpose utilizing an exponential representation of the observed line ratios used to obtain density diagnostics. Also, we have discussed the solution and criteria in order to determine scale height. Meanwhile, in Section 3, numerical studies providing an acceptable solution for our case, as well as an application has been provided to determine the referenced height.

## 2. Basic Formulations

### *2.1 Exponential Representation for Electron Density Determination in Solar Corona*

In what follows, we will use the exponential representation for the electron density used for solar corona assuming that the plasma is isothermal and of homogeneous nature[4]. In this representation the electron density varies with height $z$ according to the following equation:

$$N_e = N_o \operatorname{Exp}(-z/h),$$

By substituting $z = (r\text{-}r_o)\,R_o^2$, and $h = H_o\, r_o\, r,$ we get

$$N_e = N_o \operatorname{Exp}(-(r\text{-}r_o)\,R_o^2 / H_o\, r_o\, r), \tag{2.1}$$

where $H_o$ is the density scale height, $r$ is the distance from the surface of the sun to a place in the corona, $N_e$ is the electron density and $r_o$ is an arbitrary reference height and $N_o$ is the value of $N_e$ at $r = r_o$.

Equation (2.1) could be rewritten as

$$Y = C_1 + C_2 X\,, \tag{2.2}$$

where

$$C_1 = \ln N_o - 1 / H_o\, r_o\,, \tag{2.3.1}$$

$$C_2 = 1 / H_o\,, \tag{2.3.2}$$

$$Y = \ln N_e\,, \tag{2.3.3}$$

$$X = 1 / r. \tag{2.3.4}$$

Suppose we have measured the electron density (hence $Y$'s) in the solar corona at different values of $r$ (hence $X$'s), as obtained from the CDS spectrograph on the SOHO satellite[8]. As discussed earlier we will investigate Si X spectral lines in the coronal holes.

### *2.2 Determination of $C_1$ and $C_2$ and the Error Estimates*

*2.2.1 Solution for $C_1$ and $C_2$*

Let $m$ (say) observational data $(X_k,\, Y_k)$; $k = 1, 2, ..., m$ be known, then defining

$$A_1 = \sum_{j=1}^{m} X_j;\; A_2 = \sum_{j=1}^{m} X_j^2;\; B_1 = \sum_{j=1}^{m} Y_j;\; B_2 = \sum_{j=1}^{m} Y_j X_j;\; B_3 = \sum_{j=1}^{m} Y_j^2\;, \tag{2.4}$$

we get for the solutions of $C_1$ and $C_2$ of Equation (2.2) in the sense of the least-squares criterion the expressions

$$C_1 = (B_1A_2 - B_2A_1) / D, \tag{2.5}$$

$$C_2 = (B_2m - B_1A_1) / D, \tag{2.6}$$

where

$$D = mA_2 - A_1^2. \tag{2.7}$$

The error estimates are given in the following points.

*2.2.2. Standard Error of the Fit*

According to the least-squares criterion, the standard error of the fit is given by

$$\sigma = \left[ \sum_{j=1}^{m} \{C_1 + C_2X_j - Y_j\}^2 /(m-2) \right]^{1/2},$$

Expanding we get

$$\sigma = \left[ \{B_3 - C_1^2 m - C_1^2 A_2 - 2C_1C_2A_1\} /(m-2) \right]^{1/2}, \tag{2.8}$$

where $A_1$, $A_2$, $B_3$, $C_1$ and $C_2$ are given by Equations (2.4), (2.5) and (2.6). Note that if the precision is measured by probable error *e,* then

$$e = 0.6745\ \sigma. \tag{2.9}$$

*2.2.3. Standard Errors of $C_1$ and $C_2$*

The standard errors of the least-squares solutions $C_1$ and $C_2$ are given as

$$\sigma_{C_1} = \sigma\sqrt{g_{11}}, \quad \sigma_{C_2} = \sigma\sqrt{g_{22}},$$

where $g_{kk}$ are the diagonal elements of the inverse of the matrix $G$ used for the solutions $C_1$ and $C2$. Therefore, the standard and the probable errors of $C_1$ and $C_2$ are

$$\sigma_{C_1} = \sigma\sqrt{A_2 / D}\ ; \quad e_{C_1 = 0.6745}\ \sigma_{C_1}, \tag{2.10}$$

given as

$$\sigma_{C_2} = \sigma\sqrt{m / D}\ ; \quad e_{C_2 = 0.6745}\ \sigma_{C_2}, \tag{2.11}$$

where $A_2$, $D$ and s are given from Equations (2.4), (2.7) and (2.8).

*2.2.4. The Average Squared Distance Q*

The average squared distance $Q$ between the least-squares estimators of $C_1$ and $C_2$ and their true values is given as in Kopal and Sharaf[13].

$$Q = \sigma^2 \sum_{i=1}^{2} 1/\lambda_i \ ,$$

where $\lambda_1$, $\lambda_2$ are the eigen values of the matrix $G$. Evaluating $\lambda$'s we get

$$Q = (\sigma_{C_1})^2 + (\sigma_{C_2})^2 \ . \tag{2.12}$$

*2.2.5. Coefficient of Correlation*

Another useful estimator is the correlation coefficient which is a measure of the degree to which the two variables ($X$, $Y$) are related to each other. In our case, the coefficient of correlation $R$ is given by

$$R = C_2 \left( \frac{D}{mB_3 - B_1^2} \right)^{1/2} \ . \tag{2.13}$$

The closer $r$ is to 1 or –1 the stronger the exponential representation between the two variables ($X$, $Y$). The closer $r$ is to zero, the weaker the exponential representation. The sign of $r$ indicates the direction of the representation between $X$ and $Y$.

## 3. Numerical Study

### *3.1 Acceptable Solution Set*

Although the least-squares method is one of the most powerful techniques that could be used for the present problem, it is at the same time exceedingly critical. This is because the least-squares estimate does not have detecting and controlling techniques for the sensitivity of the solution to the optimization criterion of the variance $\sigma^2$ as minimum. As a result there may exist a situation in which there are many significantly different values of the solutions $C_1$ and $C_2$ [see Tables 1, 2] that the variance to an acceptably small value. At this stage we should point out that: (1) the accuracy of the estimators and the accuracy of the fitted representation [Equation (2.2)] are two distinct problems; and (2) an accurate estimator will always produce small variance, but small variance does not guarantee an accurate estimator.

According to these two notes, it is necessary to reformalize the concept of an *acceptable solution for our problem.* This last point is illustrated as follows. Let us define an acceptable solution set for the present problem as

$$\text{S.S.} = \{C_1, C_2 : e \le \varepsilon_{1,} \, |e_{C_1}/C_1| \le \varepsilon_{2,} \, |e_{C_2}/C_2| \le \varepsilon_3, Q \le \varepsilon_4, r \ge \varepsilon_5 \ .\} \tag{3.1}$$

The ε's are error tolerances, of course, the set of these tolerances is different for the different color indices. In this respect, the error controlling formulae of Section 2 could be useful for determining the coefficients $C_1$ and $C_2$ very accurately.

### *3.2 Numerical Results*

– The data samples were collected from[8]. The observations used here are coronal densities measured above the north and south coronal holes in 1997 May 3 and 12, respectively. The total number of points are 154. It should be noted that the ordinates of these data are divided by $10^8$ in our calculations.

– The selection criteria for the considered data is

$$|O - C| \leq 0.18 \tag{3.2}$$

Where $O$ is the observed value of $Y$, while $C$ is that calculated from Equation (2.2).

– As a result of the criteria (3.2), the number of points, is reduced to 124. The corresponding elements and the tolerances of an accurate acceptable solution set is described in Section 3.1 and are listed in Table 2.

– The dependence of the solutions and the tolerances ε's on the selection criteria are listed in Table 1. Also the error bars of Fig. 1 are given as typical examples of this dependence.

Table 1. Dependence of the solutions and the tolerances on the selection criteria.

| *O-C* | No. | $C_1$ | $C_2$ | $e$ | $eC_1/C_1$ | $eC_2/C_2$ | $Q$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| 20 | 154 | 13.3143 | 6.16678 | 0.1085 | 0.0236 | 0.0251 | 0.1012 | 0.9073 |
| 0.4 | 152 | 13.3028 | 6.18755 | 0.0981 | 0.0193 | 0.0206 | 0.0827 | 0.9237 |
| 0.3 | 146 | 13.1077 | 6.38358 | 0.0847 | 0.0149 | 0.0159 | 0.0661 | 0.9433 |
| 0.25 | 140 | 12.9552 | 6.53728 | 0.0774 | 0.0131 | 0.0138 | 0.0605 | 0.952 |
| 0.23 | 135 | 12.8421 | 6.65152 | 0.0718 | 0.012 | 0.0124 | 0.0562 | 0.9581 |
| 0.2 | 127 | 12.765 | 6.72925 | 0.0639 | 0.0098 | 0.0104 | 0.0503 | 0.9653 |
| 0.18 | 124 | 12.6357 | 6.86087 | 0.0611 | 0.0092 | 0.0097 | 0.0483 | 0.9686 |

– Figure 2 represents the calculated distribution of electron densities as a function of height together with the corresponding correlation coefficient.

### *3.3. Applications*

#### *3.3.1 Determination of the Density Scale Height*

Following Equation (2.3.2), we could calculate the density scale height, as well as using the data from Table 2, we have

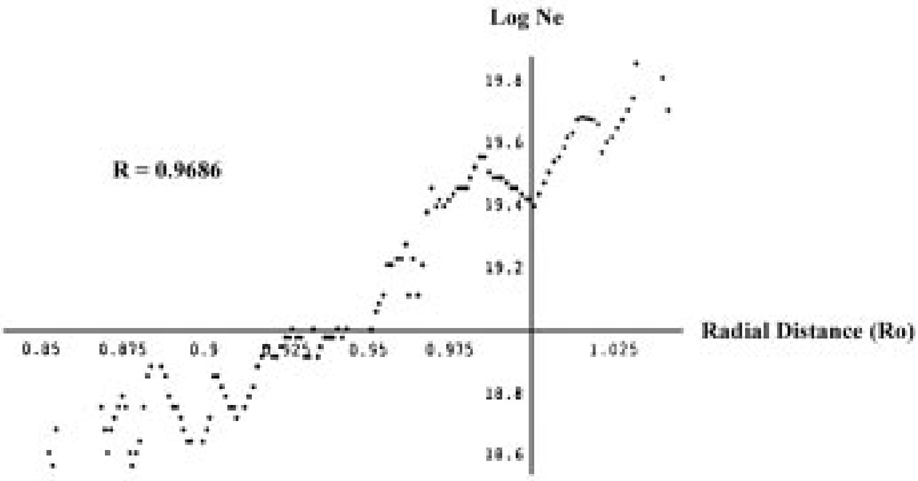


FIG. 1. Observed electron density distribution as a function of height.

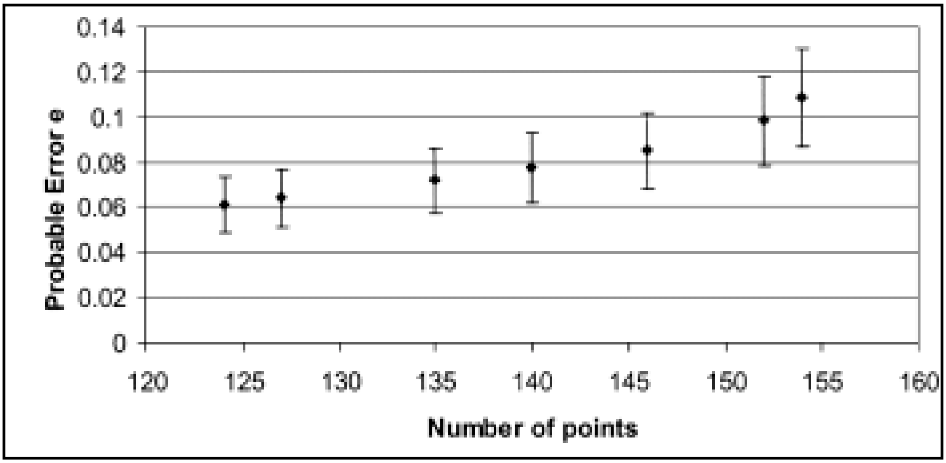


FIG. 2. Error bars are indicated for *e*.

TABLE 2. Elements of the optimum acceptable solution set.

$$C_1 = 12.6357$$
$$C_2 = 6.86087$$
$$e = 0.0611$$
$$e_{C_1}/C_1 = 8.0724 * 10^{-3}$$
$$e_{C_2}/C_2 = 0.0157$$
$$Q = 0.0483$$
$$R = 0.9686$$

$$H_o = 1.46 \times 10^7 \text{ meters},$$

Which is in good agreement with the value discussed in[8] for a value of scale height temperature $T_s$ i.e. the temperature that the plasma would have in hydrostatic equilibrium and which has the value of $2 \times 10^5$ K.

### 3.3.2. Determination of the Arbitrary Reference Height

In order to determine the value of the arbitrary reference height, as in the following equation:

$$r_o = C_2 / (\ln N_o - C_1)$$

Also if we adopt for $N_o$ the value of $3.6 \times 10^8$ cm$^{-3}$, and using the data mentioned in Table 2, we get for $r_o$ the value of 0.5.

## 4. Conclusion

In concluding the present paper, selection and solution criteria enable us to determine very accurately the density and scale height in the solar corona. From the slope of the Equation (2.2) we compute the density scale height $H_o$ with the value of $1.46 \times 10^7$ meters, which is in a good agreement with that mentioned in Fludra's work. From this slope and the coefficient $C_2$ we find also the value of the arbitrary reference height to be 0.5.

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# تشخيص تدرج الكثافة في كورونا الشمس

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*المستخلص.* لقد تناولنا في هذا البحث موضوع توزيع الإلكترونات مع البعد في الغلاف الجوي الشمسي (وخاصة طبقة الكورونا) ، وقد استخدمنا لهذه العلاقة تمثيلا ذو دلالة أسية . ومنه تم استخدام معايير حلول واختيار دقيقة جداً لتعيين تدرج الارتفاع للكثافة في تلك الطبقة من الغلاف الشمسي . وقد تم من المنحني حساب هذه القيمة لنحصل على ١,٤٦ × ١٠^٧م والتي تتفق إلى حد كبير مع ما هو متعارف عليه . كما تم أيضًا الحصول من المنحني ومن قيمة $C_2$ على قيمة الارتفاع الاختياري وقدرها ٠,٥ .